THE USE OF OVERLAPPED SUBARRAY TECHNIQUES IN SIMULTANEOUS RECEIVE BEAM LINEAR ARRAYS

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ABSTRACT

A linear array of antennas connected to an appropriate beam-forming matrix or having outputs processed digitally to yield a spatial Fourier analysis allows great flexibility for multiple-beam reception in a HF radar. However, for large numbers of elements the cost becomes unmanageable. Subarraying the elements reduces the degrees of freedom and hence the cost but introduces difficulties with grating lobe effects. A design approach is developed which uses steerable subarrays whose outputs are used to develop multiple simultaneous beams lying within the envelope of the subarray pattern. The combination of amplitude taper within the subarrays and overlap of the subarrays can be used in a systematic way to realise simultaneous beams having an equiripple side-lobe structure of specified level below the beam maximum. A design process is developed and illustrated by typical applications.

1. INTRODUCTION

The work presented in this paper was motivated by the receiving antenna requirements for an experimental HF over-the-horizon radar. The designer is required to provide a specified resolution (ie beamwidth) yet there is an upper bound on the revisit time for scanning to satisfy tracking considerations. Hence, simultaneous receive beams are necessary. The long wavelengths at HF (typically tens of metres) mean that an array rather than a reflector antenna must be used and clearly electronic steering is mandatory. The costs of real estate, site works and interconnecting cables, coupled with the difficulty in realising an array aperture with significant projected height, mean that it is usually necessary to settle for a linear rather than a planar array. The beam-forming and -steering problem then becomes one in a single steer dimension, expressed in the following in terms of θ, the angle measured from array broadside.

The simultaneous receive beams may be realised from a set of antenna outputs either by some form of hardware beamformer or by digital processing of the digitised outputs to provide a spatial Fourier transform. The operating frequency bandwidth is typically wide for HF radars and a large number of array elements is required to give a sufficiently-filled array satisfying the resolution requirements. Hence a hardware multiple beam-former covering the whole surveillance sector is impracticable. The approach via digitising of array outputs and spatial Fourier transformation to synthesise beams covering the whole surveillance sector is conceptually attractive but costly: if there are several hundred array elements spread over an array length of some kilometres the problems of cabling each element via a suitable receiver/preamplifier to a central point whilst ensuring that the cables retain their phase calibration are such as to make this an economically unattractive approach.

To contain costs, whilst allowing either a hardware or software synthesis of a reasonably small number of beams, it is necessary to reduce the number of inputs to the beamformer. A useful approach, then, is to organize the array into subarrays and cable back to the central beamformer the relatively small number of subarray outputs which provide inputs for the simultaneous beamformer. The individual subarrays need to be steerable to provide a coarse sector selection within which the beamformer then forms a cluster of simultaneous beams. With appropriate control of the subarray steering in coördination with the synthesis of the simultaneous beams, the resultant beams can be steered as a cluster anywhere in the surveillance sector.

At first sight, it appears that this approach overcomes the major difficulties in realising a steerable cluster of simultaneous beams without any of the disadvantages of other approaches. However, it is shown in the following that grating lobes of the synthesised beams limit the success of the technique and require special methods to be adopted, involving overlapping the subarrays, if sidelobe performance is to be acceptable.

2. THE NEED FOR SUBARRAY OVERLAP

Figure 1 shows an example of the approach outlined in the previous section for 8 subarrays having 16 elements per subarray. No weighting is applied within the subarrays or to the subarray outputs. The figure shows how the synthesised beam may be realised as a product of the subarray pattern and the array factor. The array factor is the pattern of an array of 8 isotropes spaced by the subarray spacing. In figure 1 the subarray steer direction coincides with the direction of the synthesised beam. In this case, the grating lobes of the array factor all fall exactly on nulls of the subarray pattern so that they are effectively cancelled. In this case an ideal, linear phase progression is set up across the array so the pattern is clearly that of a correctly-phased 8 x 16 = 128 element array.

Figure 2 illustrates the synthesis of a beam away from the direction of steer of the subarray. In this case, the array factor slides, in sin θ space, relative to the subarray pattern to provide the beam offset and in so doing causes the array factor grating lobes to move into areas of sin θ space away from the subarray pattern nulls. In the example shown in figure 2 the resultant beam pattern formed by the product of the subarray pattern and array factor would have two lobes of nearly-equal amplitude and high sidelobes. Although a specific example has been chosen to illustrate the effect it is clear that the behaviour illustrated is general: for non-weighted, non-overlapped subarrays the distance, in sin θ space, between the peak of the subarray pattern and its first null is equal to the spacing of the array factor grating lobes. Thus, moving the grating lobe structure relative to the subarray pattern in order to realise a beam offset from the subarray steer direction inevitably causes a grating lobe to enter the subarray pattern main beam.

A way out of this is to overlap subarrays. This leaves the subarray pattern unchanged but the array factor grating-lobe spacing is increased. Figure 3 shows an example based on that used in figures 1 and 2 except that the subarray spacing is effectively halved by the subarray overlap illustrated. Clearly, the first grating lobe is now well clear of the subarray pattern main lobe so that sliding the array factor to produce a synthesised beam away from the subarray steered direction will not cause this grating lobe to enter the subarray pattern main lobe. However, if the array factor grating lobes coincide with sidelobe peaks of the subarray pattern the sidelobe performance of the synthesised beam may not be acceptable. A solution to this problem is to employ amplitude tapering within the individual subarrays to reduce the sidelobes of the subarray pattern. This will cause some broadening of the subarray pattern main beam. The design process then seeks to arrive at a minimum amount of subarray overlap (the case illustrated in figure 3 may be designated "50% overlap") which will allow simultaneous beams to be synthesised within a specified Sector of the amplitude-tapered subarray pattern, with all sidelobes below a specified level.

3. THE GENERAL SUBJECT OF SUBARRAY OVERLAP FOR PATTERN SYNTHESIS

There is an established literature, reflecting a well-developed methodology, for so-called "overlapped subarrayed scanning antennas" [1 2 3]. This work has as its origins the search for economy in limited-scan reflector antennas[1]. A reflector is used to effectively "magnify", using optical principles, the aperture of a relatively small scanned array used as a feed. As developed by Fante[3] the concept is confined to neither limited-scan nor reflector antennas. The concept is to synthesise a subarray pattern which is an approximation to a rectangular pattern and overlay this with a grating lobe series representing the array factor: again, the product of the two patterns results in a single-lobed pattern which can be placed anywhere (either simultaneously or sequentially) within the flat-topped subarray beam, so long as the desired

relationship between subarray pattern width and grating-lobe spacing is chosen appropriately. To realise the flat-topped subarray pattern, an approximation to a rectangular response, requires sin x/x type subarray weighting and to obtain a reasonable approximation a large proportion (if not all) of the available array aperture is used by each "subarray". Such a feed arrangement is readily implemented using a lens in an optical type of feed; it is not practicable in an HF array where all elements are connected to the beamforming equipment by cables. In this latter case, which is the case we confine attention to, the amount of overlap must be minimised: overlapping means that an array element is used more than once so that it must have its output split and be cabled to 2 or more sets of beamforming hardware. Further, HF reception is normally externally noise-limited so that the need for a flat-topped subarray pattern is not pressing. So long as the sidelobe level relative to the main beam of the synthesised patterns is maintained, electronic gain can be used to equalise the beam responses.

4. BASIS OF THE DESIGN APPROACH

4.1 Specification and selection of basic array parameters

The specification to be provided must give the following.

a. Instantaneous surveillance sector, $\pm\ \theta_0$: This is the angular coverage of the subarray pattern within which the simultaneous beams will be developed. θ is measured from the array broadside and $\pm\ \theta_0$ is the sector width with the subarrays scanned to broadside. (All beams broaden by a factor $1/\cos\theta$ as the array is scanned).

b. Scan limit for extreme beams of cluster, $\pm\ \theta_{max}$. This marks the extreme ends of the surveillance sector.

c. Allowable taper of simultaneous beam amplitudes caused by the subarray pattern, Q dB: Q is expressed as a positive dB - it is the ratio of the simultaneous beam peaks at $\theta=0$ and $\theta=\theta_0$ with the subarrays steered to broadside.

d. Minimum suppression of sidelobes relative to the main lobe of the simultaneous beams, R dB: R is expressed as a positive dB.

e. Beamwidth of the simultaneous beams at broadside, θ_b.

From these specifications the array design proceeds initially along conventional lines as follows.

Total array length: The array length, L, is given by,

$$L/\lambda = 50.9\ F/\theta_b \tag{1}$$

if θ_b is in degrees, λ is the wavelength and the factor F is the "beam-broadening factor" consequent upon an assumed Chebychev weighting of subarray outputs to produce the specified sidelobe suppression. From the formula (50) of Stegen[4] it can be shown that a good approximation is

$$F = 0.716\sqrt{0.360 + 0.693\ \ln qr + (\ln qr)/2q^2r^2} \tag{2}$$

where $R = 20\ \log_{10} r$, $Q = 20 \log_{10} q$

Element spacing and total number of elements

The condition for a fully-formed grating lobe just entering visible space (at end-fire) when the array is scanned to the scan limit, θ_{max}, is that the inter-element spacing, d, be given by

$$d = \lambda/(1+\sin\ \theta_{max}) \tag{3}$$

This represents an upper bound on d; in practice some factor of safety may be desirable to reduce d slightly so that the grating lobe remains entirely outside visible space. Alternatively, the element factor may have a null in the end-fire direction (eg, for vertical loop elements) so that d may be allowed to be a little larger than given by (3). In any case, a value for d may be derived and then the total number of array elements is about L/d.

4.2 Subarray parameters

The subarray parameters must now be defined from the specification of θ_0, Q and R. The subarray pattern is taken to be an equiripple Chebychev representation of the form[5]

$$T_{N-1}(z_0 \cos u/2) \qquad (4)$$

where $u=kd \sin\theta$, $k=2\pi/\lambda$, N is the number of elements in the subarray and z_0 can be found from the requirement that sidelobes must be suppressed by R dB for all synthesised beams. The worst case is for the beam maximum to be down Q dB because of the subarray pattern and an array factor grating lobe to be at a value of u for which the subarray pattern is at the level of its sidelobe peaks. To guarantee a sidelobe suppression of R dB, the subarray pattern must accordingly be designed for sidelobes R+Q dB below the main beam.

Thus

$$T_{N-1}(z_0) = qr \qquad (5)$$

Now the subarray pattern must be down Q dB at $u_0 = kd\sin\theta_0$, so as a second condition we have,

$$T_{N-1}(z_0 \cos u_0/2) = r \qquad (6)$$

Equations (5) and (6) may be expressed in terms of the cosh function representation of Chebychev polynomials and then solved iteratively for z_0 and N,

$$\cosh([N-1]\cosh^{-1} z_0) = qr \qquad (7)$$

$$\cosh([N-1]\cosh^{-1}(z_0 \cos u_0/2)) = r \qquad (8)$$

Figure 4 is a design chart, based on these equations, for the determination of N given u_0 and R+Q for the specific case of $Q = 3 + R$ and will be used in the examples to be given in Section 5.

4.3 Subarray overlap

The final step in the design process is to determine the amount of overlap. From the discussion in Section 2 it is clear that the overlap needs to be such that the closest-in grating lobe just enters the subarray main beam, at the - (Q+R) dB level, for the array factor placed at $\theta=\theta_0$. Figure 5 illustrates the criterion: for Q+R = 20 dB (the dashed subarray pattern) with the array factor at A, grating lobe responses are at A', A'' and A'''. For Q+R = 35 dB (the solid subarray pattern) with the array factor at B, grating lobe responses are at B' and B''. The grating lobe separations, respectively ΔU_{20} and ΔU_{35} as shown in figure 5, determine the subarray centre-to-centre spacing and hence the overlap. Since $\Delta U_{35} > \Delta U_{20}$ the subarray spacing is less (overlap is more) for the 35 dB design than for the 20 dB case, as would be expected on intuitive grounds.

Since the array factor is expressible as

$$\sum_{n=1}^{N} a_n \exp(jnS[u-u'] \tag{9}$$

where a_n is the weighting used in synthesising the array factor, S is the subarray separation in multiples of d and $u' = kd\sin\theta'$ where θ' is the steer angle, it is clear that the first grating lobe occurs at

$$M[u-u'] = 2\pi \tag{10}$$

But $u-u' = \Delta u$ in figure 5, for $u' = u_0$, so the subarray separation, S, in multiples of d is given by,

$$S = \frac{2\pi}{\Delta u} \tag{11}$$

The value of Δu may be derived by noting that

$$\Delta u = u_0 + u_1 \tag{12}$$

where

$$T_{N-1}(z_0 \cos u_1/2) = 1 \tag{13}$$

Equation (12) has a multiplicity of solution; the smallest value of u_1 which satisfies the equation is required and can be shown to be given by

$$u_1 = 2\cos^{-1}(1/z_0) \tag{14}$$

This completes the design process, but some iteration may be necessary to effect minor compromises. S must be made an integer and normally the designer would choose the next smallest integer below that given by equation (11). Figure 6 is the result of the design process, giving integral values of S for subarray sizes in the range 4 to 64 for various values of R+Q, for Q = 3 dB. A similar chart could be produced for other values of Q but this one is included so it can be used in the examples which follow.

It is of interest to note that the condition of S = N/2 (50% overlap) as realised by the example of figure 3, produces a value of R+Q in the range 30-35 dB for N in the range 8 to 64, and beyond, for Q = 3 dB, as shown by the dotted line in figure 6. This is an interesting result, as going beyond a 50% overlap (S < N/2) involves nesting subarrays 3 or more deep, and represents a step increase in complexity and cost.

5. EXAMPLES OF THE DESIGN APPROACH

Two examples are given to illustrate the design steps set out in the previous Section. The examples are synthetic in that the specifications are selected so that the charts of figures 4 and 6 may be used but are nevertheless illustrative of the process. Specifications for the two examples are given below.

Specification	Example 1	Example 2
Instantaneous surveillance sector, $\pm\theta_0$	$\pm 5°$ (ie 10° beamwidth of instantaneous surveillance)	$\pm 15°$ (ie 3° beamwidth of instantaneous surveillance)
Scan limit for extreme beams of cluster $\pm\theta_{max}$	$\pm 50°$ (ie 100° surveillance sector)	$\pm 30°$ (ie 60° surveillance sector)
Allowable taper of beam cluster arising from subarray pattern, Q dB	3 dB	3 dB
Minimum sidelobe suppression, R dB	20 dB	29 dB
Beamwidth of synthesised beams, θ_b	0.5° (20 simultaneous beams intersecting at 3 dB points)	0.5° (6 simultaneous beams intersecting at 3 dB points)

5.1 Example 1

For R = 20 dB, r = 10.
For Q = 3 dB, q = 1.41.
Hence, equation (2) gives F = 1.063.
Then (1), with the specified θ_b gives L/λ = 108.2.

Equation (3) gives d = 0.566λ for a grating lobe just entering visible space. We postulate elements with some small endfire discrimination and relax d to 0.58λ. For the calculated value of L this results in approximately 187 elements. Then $u_0 = kd\sin\theta_0 = 0.317$. Figure 4 may now be used to determine N (the figure is drawn for Q = 3). The coordinates R+Q = 23, u_0 = 0.317 lie close to the N = 10 line. Finally, figure 6, again drawn for Q = 3, determines that for N = 10, R+Q = 23, S = 6 is just sufficient separation. Whether by logic or by brute force laying out of the elements it can be shown that this array design calls for an array of 30 or 31 subarrays of 10 elements, giving respectively 184 or 190 elements total.

5.2 Example 2

Going through similar calculations, (2) gives F = 1.222 and as a consequence (1) gives $L/\lambda = 124.4$. Equation (3) gives $d = 0.667\lambda$ as the grating lobe condition. In this case we postulate azimuthally omnidirectional elements so set $d = 0.65\lambda$ to depress the grating lobe at endfire. For the calculated value of L, this results in approximately 191 elements. Then $u_0 = kd\sin\theta_0 = 0.107$. Figure 4 indicates that N = 32 is required and figure 6 indicates S = 16 provides the specified sidelobe performance. The array realisation thus consists of 11 subarrays of 32 elements giving a total of 192 elements.

In comparing the examples it can be seen that they both realise the same 0.5° beam resolution but example 1 covers a larger instantaneous surveillance sector with simultaneous beams than does example 2. Thus, example 1 is realised by a comparatively large number of small subarrays, example 2 by a smaller number of larger subarrays.

6. CONCLUSION

A general approach to the design of subarrayed linear antenna arrays has been presented. The approach has application to HF radar receiving array design but is also appropriate to other bands where array elements are of small aperture and connected to a central beamformer by cabling. A major design aim, which is implicit in the reported approach, is the minimisation of subarray overlap, with its consequent increase in complexity and cost.

Examples have been given which illustrate the design approach.

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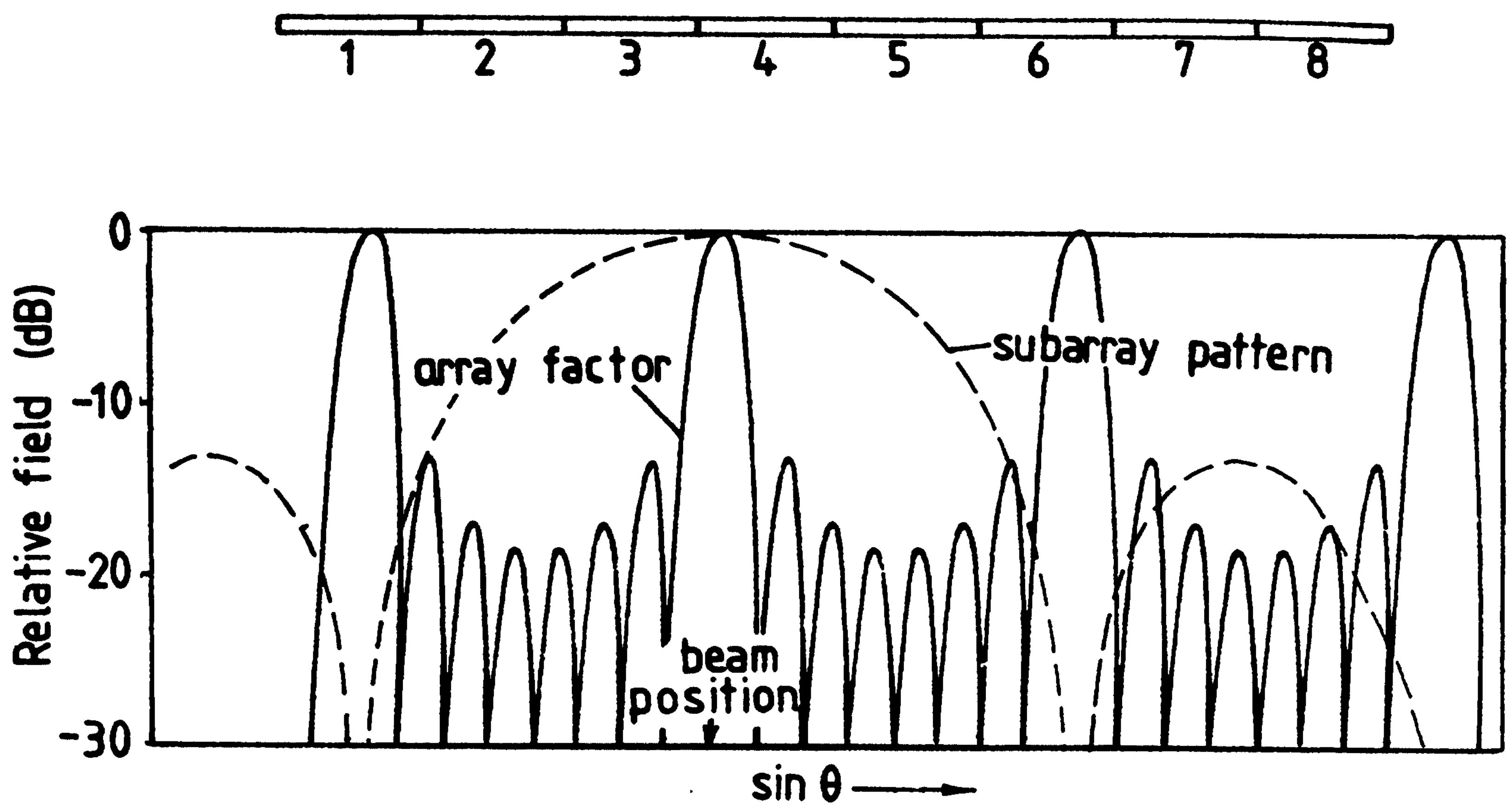

Figure 1

Effect of Array Factor Grating Lobes -
Beam Position Aligned with Subarray Pattern Maximum
(8 Subarrays of 16 Elements)

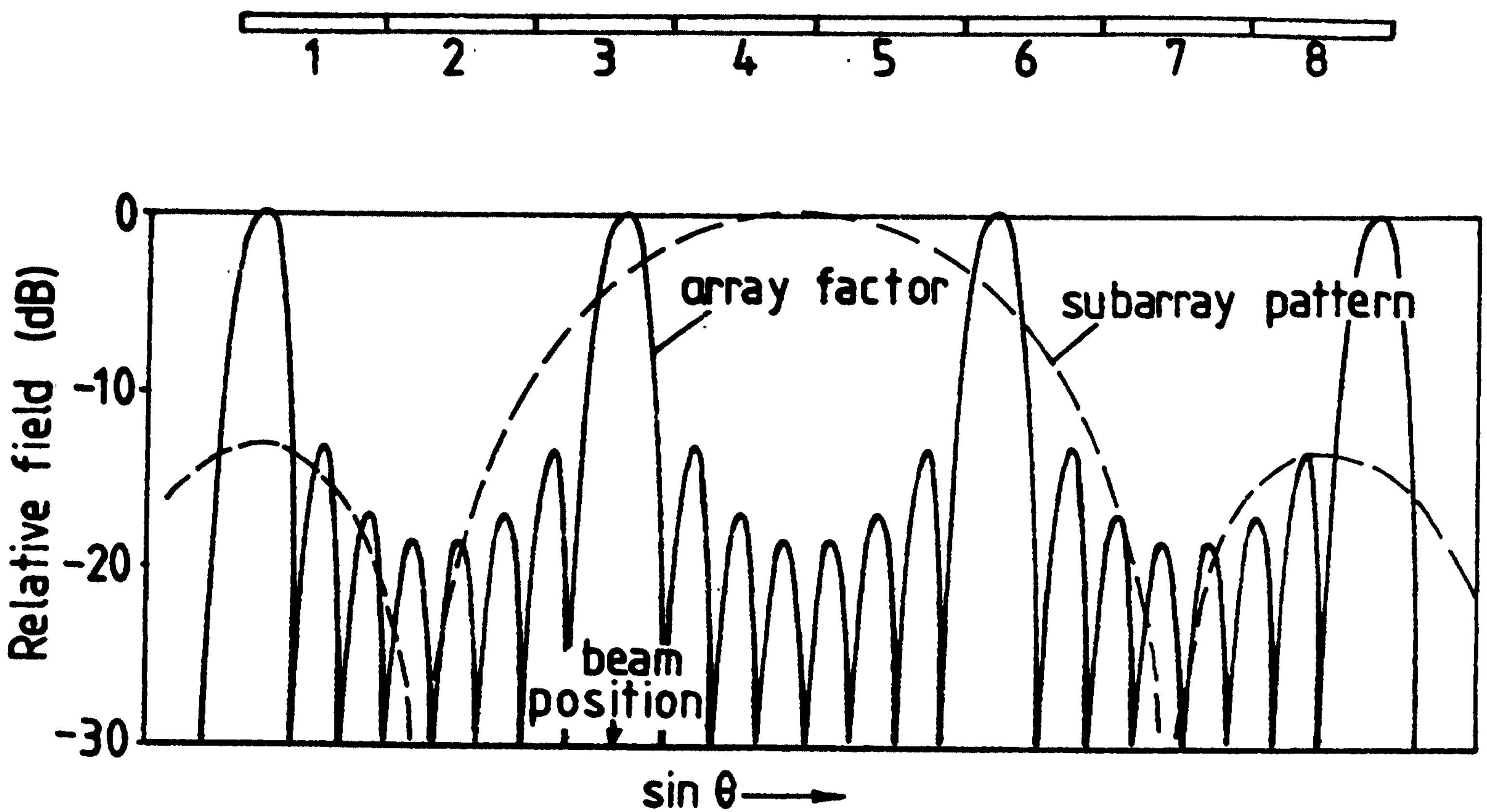

Figure 2

Effect of Array Factor Grating Lobes -
Beam Position away from Subarray Pattern Maximum
(8 Subarrays of 16 Elements)

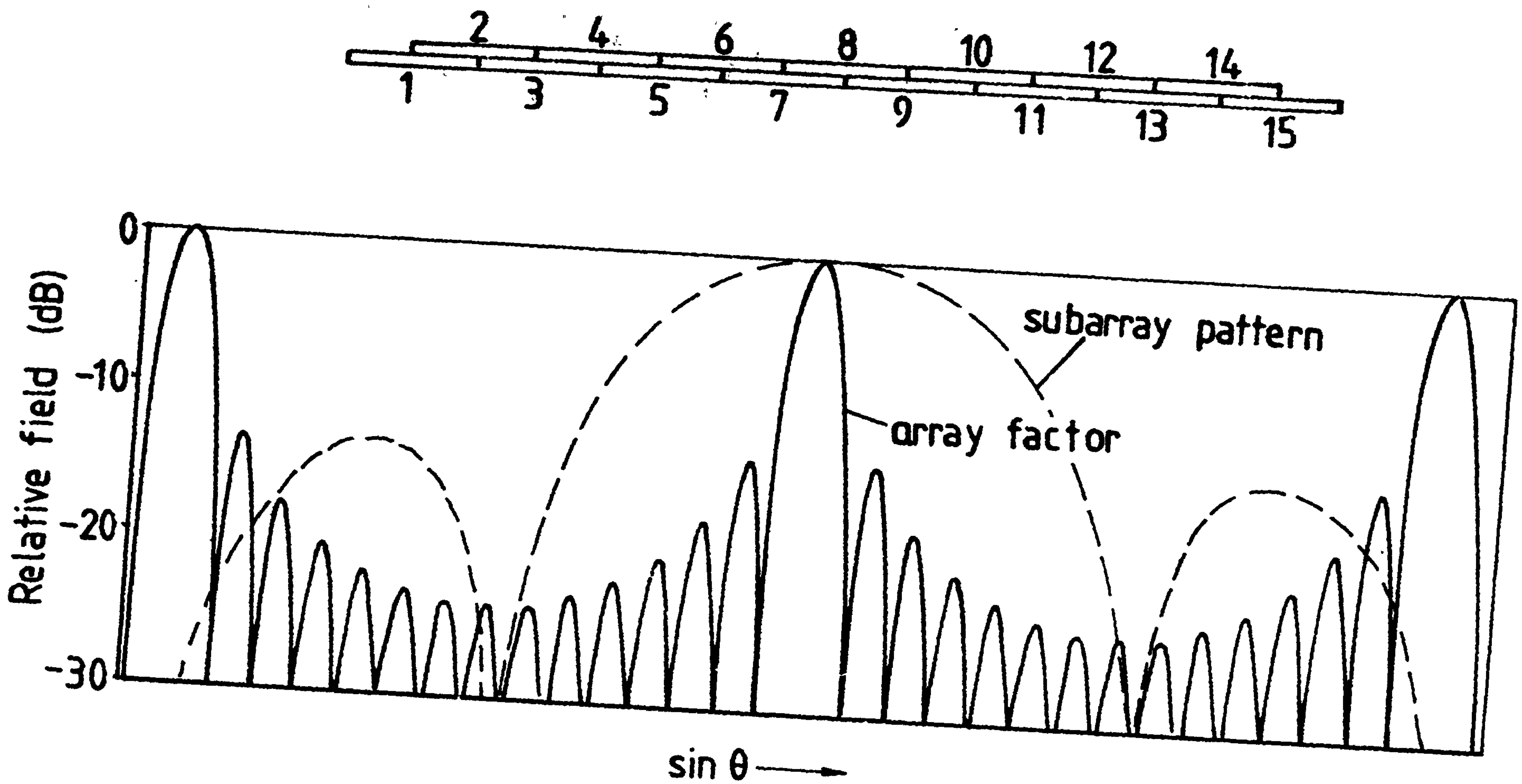

Figure 3

Array Factor and Subarray Pattern for 50% Subarray Overlap
(15 Subarrays, 16 Elements per Subarray)

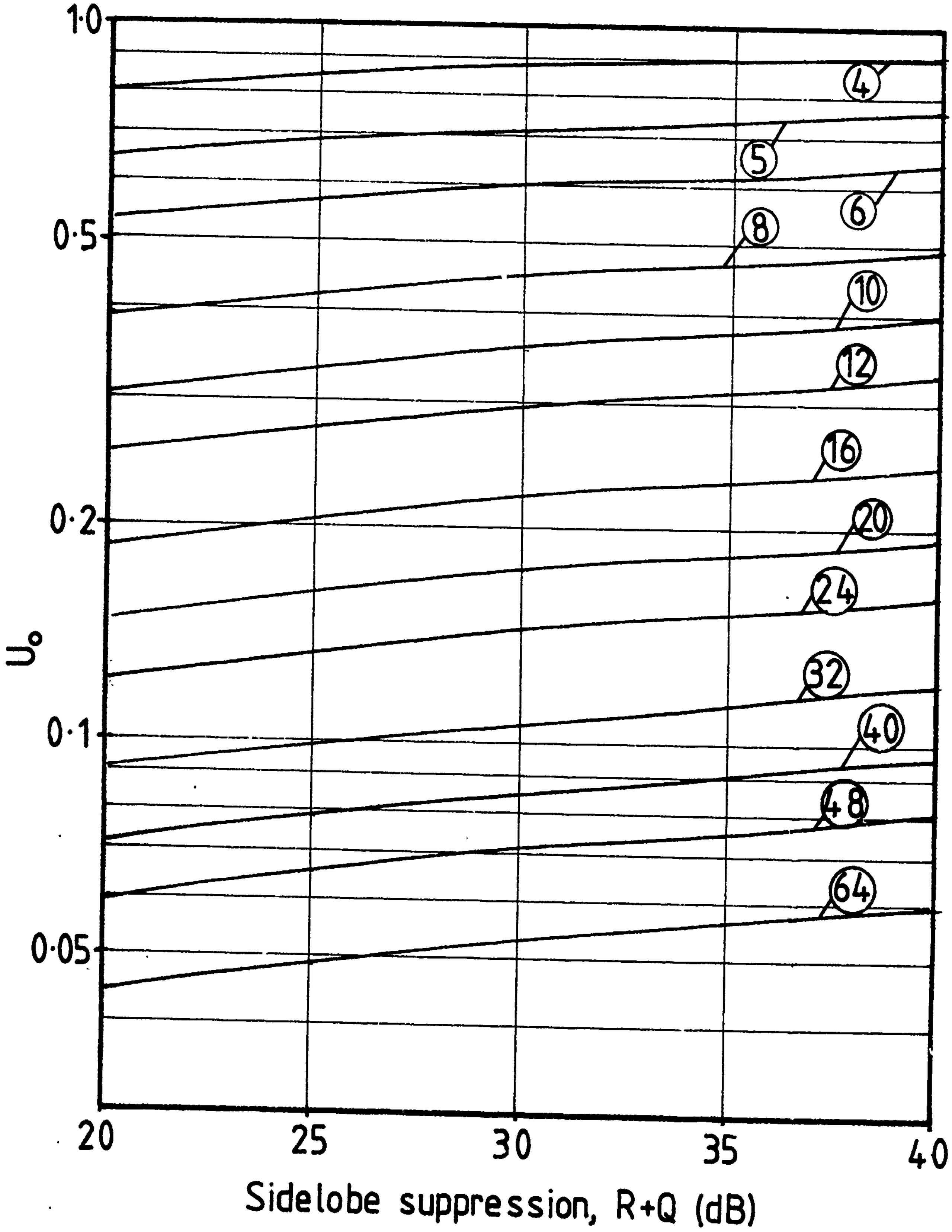

Figure 4

Design Chart for Determination of Subarray Size
(Circled Figures are Values of N, Q = 3 dB)

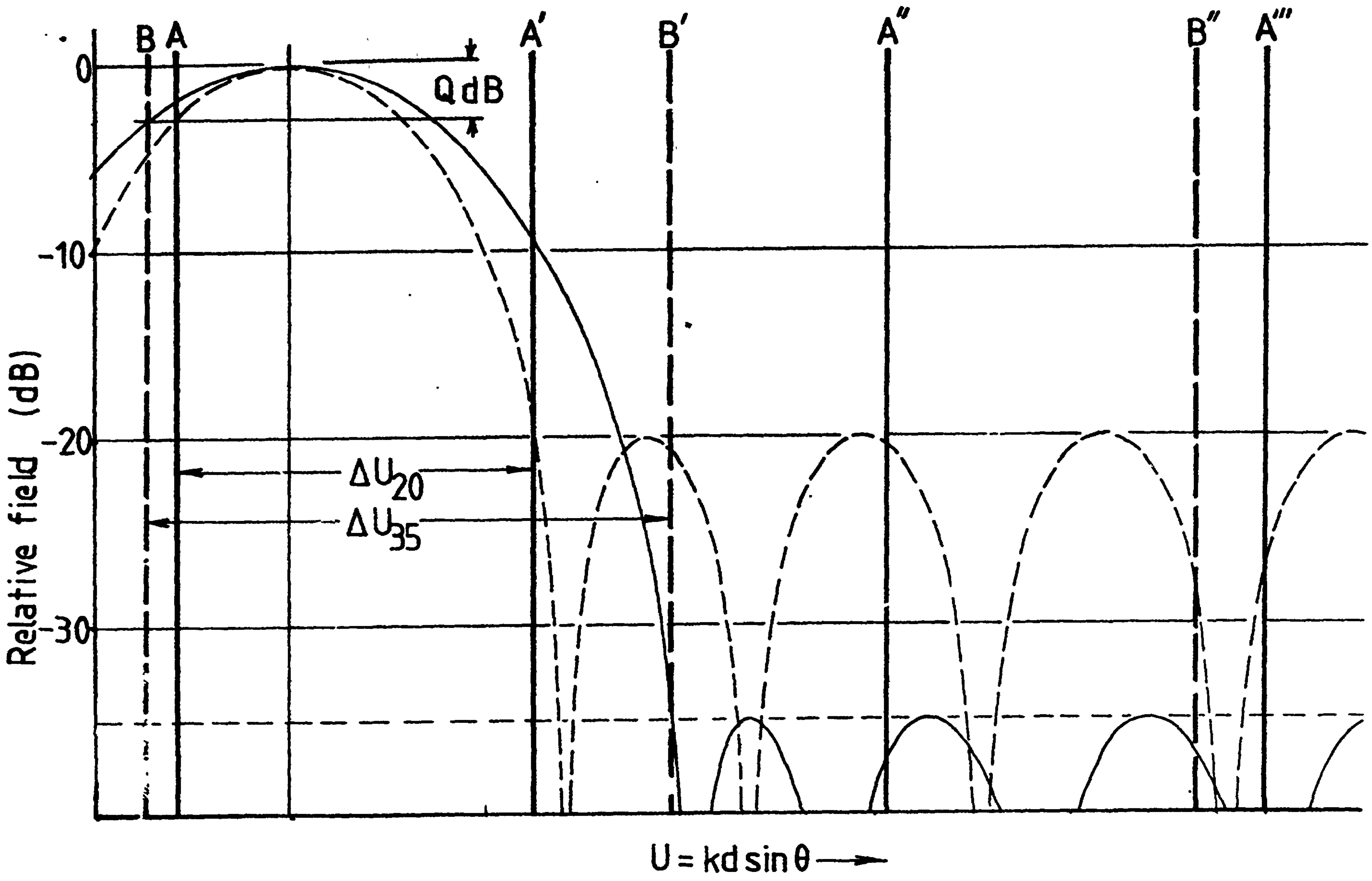

Figure 5

Illustration of Grating Lobe Placement

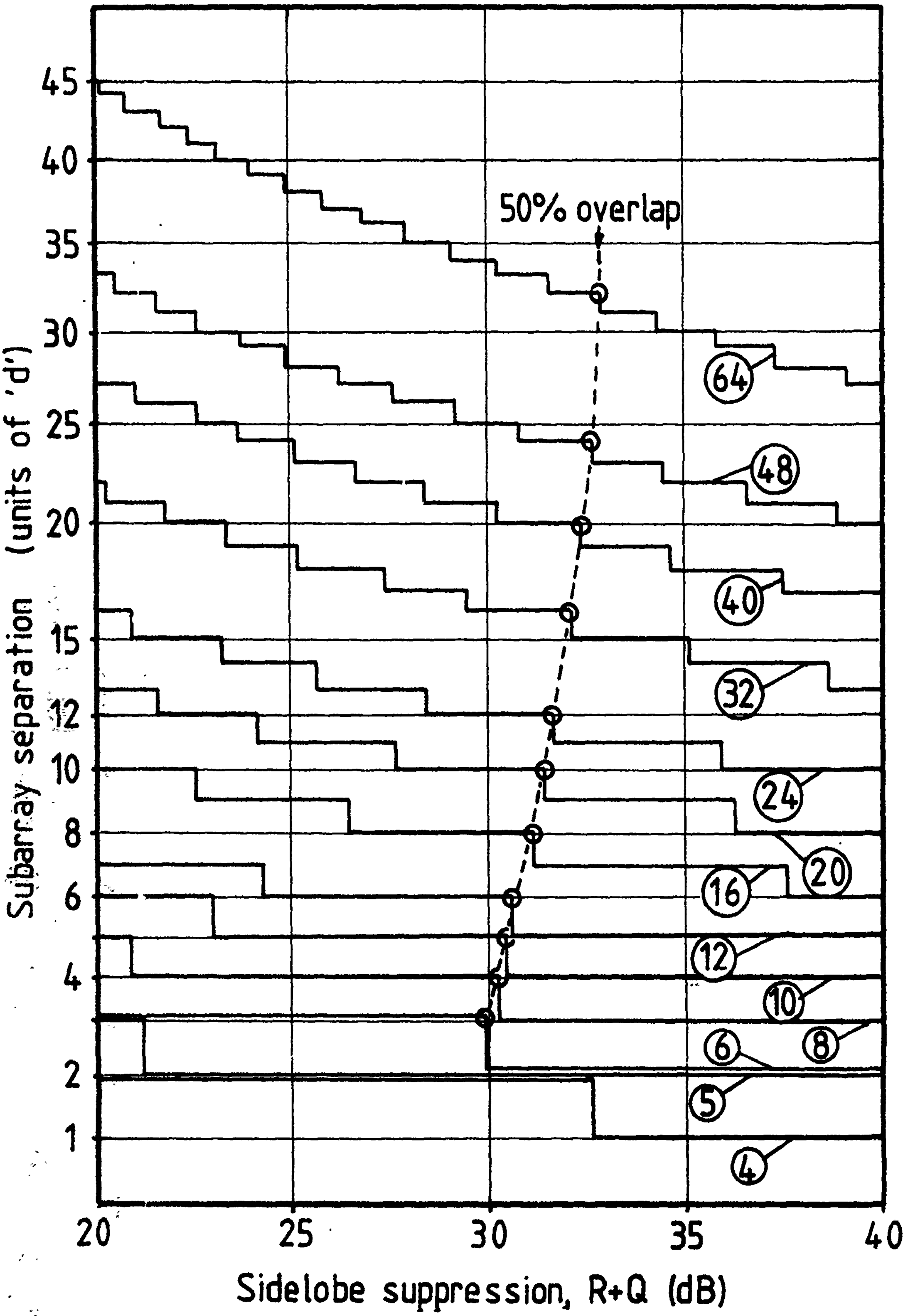

Figure 6

Subarray Separation Requirements
(Circled Figures are Values of N, Q = 3 dB)

Topic(s): Phased Arrays; Array Analysis and Design; Multiple Function Antennas.

Title: "Computer Analysis of Phased Array Far-Field Patterns for Non-Symmetrical Feed Networks"

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Abstract: A large number of non-symmetrical binary and non-binary feed networks are described, and far-field pattern analysis using a digital computer program reveals a low sidelobe structure with favorable directivity characteristics. These networks are e.g. useful for modularized active arrays, where each element amplifier consists of a group of basic modules, thus eliminating the need for special amplitude weighting and maximizing amplifier efficiency with a minimum number of component types. The technique allows broadband, multi-function operation of a single aperture. Typical active phased arrays with up to 144 elements in broadband counter-measure applications are described.

The paper will include complete feed network circuit diagrams, with at least one amplitude taper per element number and several amplitude tapers for selected linear array sizes. The calculated far-field patterns are presented and active array module constraints are tabulated.